

# VERIFICATION AND VALIDATION OF SIMULATION MODELS UNDER UNCERTAINTY

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This paper presents methodologies for validating computational models under physical, informational and model uncertainties. Integrated computational and test-based methods are being investigated at Vanderbilt University, under funding from Sandia National Laboratories, NASA, and NSF, to incorporate all three types of uncertainty for design and certification analyses of complex engineering systems. The test-only based approach is very expensive and does not make use of available analytical models of system behavior, failure modes and sensitivities. Inexpensive modeling and simulation-based methods are able to use such information. However, with the approximations in the computational models and the limited amount of statistical data on the input variables, it is difficult to associate a high degree of confidence with prediction based only on computational methods. Therefore, this paper will discuss strategies to integrate both computational and empirical methods in order to achieve both cost efficiency and high confidence. The simulation models that we address in this paper are primarily finite element-based structural analysis and limit state-based reliability prediction models.

Verification and validation under uncertainty involves quantifying the error in the model prediction and effectively comparing the prediction with the experimental result when both prediction and test data are stochastic. Several deterministic *a posteriori* error estimates are available in the literature for adaptive mesh refinement and model verification in finite element analysis. This paper presents a method to estimate the statistical distribution of discretization error in the prediction of finite element-based computational models. A collocation-based stochastic response surface method is developed for computational efficiency in predicting the stochastic distribution of error. Next, a Bayesian methodology is developed for model validation. The prior distribution of error in predicting the response is first computed, which is then updated based on experimental observation using Bayesian analysis. The prior and posterior error distributions are used to compute a validation metric that judges the validity of model prediction.

Several components of computational prediction error, such as discretization error, element error, and stochastic analysis error are included. Two types of measurement error are included, in the context of model validation: error in the measurement of input variables that affects the model prediction, and error in the measurement of output variables. The concept of Bayesian hypothesis testing is extended to system-level problems where full-scale testing is impossible. Component-level validation results are used to derive a system-level validation measure. This derivation depends on the knowledge of inter-relationships between component modules. Bayes networks are used for the propagation of validation information from the component-level to system-level. The computational methods are illustrated with several numerical examples.

## References

[1] Roache, P. J., *Verification and Validation in Computational Science and Engineering*, Hermosa Publishers, Albuquerque, August, 1998.